

# Spin Hall Effect in a Diffusive Rashba Two-dimensional Electron Gas

S. Y. Liu and X. L. Lei

Department of Physics, Shanghai Jiaotong University, 1954 Huashan Road, Shanghai 200030, China

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A nonequilibrium Green's functions approach to spin-Hall effect is developed in a diffusive two-dimensional electron system with Rashba spin-orbit interaction. In the presence of long-range impurities, the coupled quantum kinetic equations are solved analytically in the self-consistent Born approximation. It is shown that the intrinsic spin-Hall effect stems from the dc-field-induced perturbation of the density of states. In addition, there is an additional disorder-mediated process, which involves the transition of nonequilibrium electrons between two spin-orbit-coupled bands. It results in an additional collision-independent spin-Hall conductivity and leads to the vanishing of the total spin-Hall current even at nonzero temperature.

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The discovery of spin-Hall effect, namely, the nonvanishing of spin current along the direction perpendicular to the dc electric field, has attracted much recent interest in the spin-dependent transport in two-dimensional (2D) semiconductors with the spin-orbit (SO) coupling. The early studies on this issue have been devoted to the spin-orbit interaction between electrons and impurities,<sup>1,2</sup> and therefore, the spin-Hall effect exhibits the *extrinsic* character. More recently, the scattering-independent *intrinsic* spin-Hall effect, which entirely stems from the internal-field-induced spin-orbit coupling, has been predicted in the hole-doped semiconductors<sup>3</sup> and two-dimensional semiconductors with Rashba<sup>4</sup> and Dresselhaus SO coupling.<sup>5</sup>

In clean two-dimensional electron systems (2DES) with Rashba SO interaction, Sinova *et al.* have shown that the spin-Hall conductivity,  $\sigma_{sH}$ , has a universal value  $e/8\pi$  at zero temperature.<sup>4</sup> Subsequently, a great deal of works has been focused on the effect of disorder on the spin-Hall current. When the two-dimensional systems are sufficiently dirty and the Anderson localization is dominant, the spin-Hall conductivity is found to be independent of the disorder.<sup>6</sup> In diffusion regime, the studies have been carried out when the presence of short-range disorders. It has been demonstrated in Ref. 7,8,9 that the collision broadening can reduce the spin-Hall current and the ballistic value of  $\sigma_{sH}$  can only be held for weak disorder. However, the further investigation revealed that the persistent spin-hall conductivity should be completely suppressed. This conclusion has been obtained by different methods, such as the Kubo formula,<sup>10,11,12</sup> the Keldysh formalism,<sup>13</sup> and spin-density method<sup>14</sup> *etc.* It is commonly accepted that such complete cancellation of spin-Hall current is not due to any symmetry<sup>15</sup> and closely relates to the isotropy of the electron-impurity scattering.<sup>12</sup>

However, for the realistic 2D systems, it is well known that the disorder collision being short-range is a crude approximation and the electron-impurity scattering is always long-ranged. In Ref. 12, by considering the effect of long-range disorders on the spin-Hall conductivity in 2D electron systems with Rashba spin-orbit coupling, the nonvanishing of spin-Hall effect has been shown due to

the anisotropy of the collision. This argument does not take the full effect of collision anisotropy, however and hence a more careful investigation should be performed.<sup>12</sup> In this letter, we construct a nonequilibrium Green's functions approach to the spin-Hall effect in the diffusive Rashba two-dimensional electrons. When the presence of the long-range impurity-electron scattering, the derived kinetic equations are solved analytically. We clarify that the dc-field-induced intrinsic spin-Hall current is completely cancelled by the spin-Hall effect, stemming from the disorder-induced transition of the perturbative electrons, and in result, the total collision-independent spin-Hall effect also vanishes when the presence of long-range disorders.

We consider a quasi-2D electron semiconductor in the  $x-y$  plane subjected to the Rashba SO interaction. The single-particle Hamiltonian of system can be written as

$$\hat{h} = \frac{\mathbf{p}^2}{2m} + \alpha \mathbf{p} \cdot (\mathbf{n} \times \vec{\sigma}), \quad (1)$$

where  $\alpha$  is the Rashba coupling constant,  $\vec{\sigma}$  are the Pauli matrices,  $m$  is the electron effective mass, and  $\mathbf{n}$  is the unit vector perpendicular to the 2DES plane. By the local spinor unitary transformation  $\hat{U}(\mathbf{p})$ <sup>11</sup>

$$\hat{U}(\mathbf{p}) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ ie^{i\varphi_{\mathbf{p}}} & -ie^{i\varphi_{\mathbf{p}}} \end{pmatrix}, \quad (2)$$

the Hamiltonian (1) can be diagonalized,  $\hat{h}'(\mathbf{p}) = \hat{U}^+(\mathbf{p})\hat{h}(\mathbf{p})\hat{U}(\mathbf{p}) = \text{diag}(\varepsilon_1(p), \varepsilon_2(p))$  with  $\varepsilon_{\mu}(p) = \mathbf{p}^2/2m + (-1)^{\mu}\alpha p$  and  $\mu = 1, 2$ .

The nonequilibrium Green's functions  $\hat{G}^{r,<}(\mathbf{p}, \omega)$  for the 2DES with Rashba coupling are defined in the general manner and become  $2 \times 2$  matrices in spin space.<sup>16</sup> Obviously, in the helicity basis, where the single Hamiltonian is diagonal, the noninteracting Green's functions are also diagonal

$$\hat{G}_0^r(\mathbf{p}, \omega) = \text{diag} \left( \frac{1}{\omega - \varepsilon_1(p) + i\varepsilon}, \frac{1}{\omega - \varepsilon_2(p) + i\varepsilon} \right), \quad (3)$$

$$\hat{G}_0^<(\mathbf{p}, \omega) = -2in_F(\omega)\text{Im}\hat{G}_0^r(\mathbf{p}, \omega). \quad (4)$$

Here  $n_F(\omega)$  is the Fermi function.

In quasi-2D semiconductors, the electrons should experience scattering by impurities. The previous studies have only treated the short-range interaction between electrons and impurities and the relaxation is described by an isotropic parameter  $\tau$ . However, when the presence of two spin-orbit-coupled bands, such a simplification loses many important relaxation times, namely, the longitudinal and transverse times. In this letter, we study more realistic long-range collision and distinguish these different scattering times carefully.

In the spin basis, the electron-impurity scattering process is described through the long-range potential

$V(\mathbf{p} - \mathbf{k})$ . By transforming to the helicity basis it becomes  $\hat{T}(\mathbf{p}, \mathbf{k}) = \hat{U}^+(\mathbf{p})V(\mathbf{p} - \mathbf{k})\hat{U}(\mathbf{k})$ . To deal with the electron-impurity scattering, we restrict ourselves to the self-consistent Born approximation, which corresponds to the vertex corrections in the scheme of Kubo formalism. Hence, the self-energies can take the form

$$\hat{\Sigma}^{r,<}(\mathbf{p}, \omega) = n_i \sum_{\mathbf{k}} \hat{T}(\mathbf{p}, \mathbf{k}) \hat{G}^{r,<}(\mathbf{k}, \omega) \hat{T}^+(\mathbf{p}, \mathbf{k}), \quad (5)$$

with the impurity density  $n_i$ . Putting the matrix  $\hat{U}$  into Eq. (5) gives

$$\Sigma^{r,<}(\mathbf{p}, T, t) = \frac{1}{2} n_i \sum_{\mathbf{k}} |V(\mathbf{p} - \mathbf{k})|^2 \{a_1 G^{r,<} + a_2 \sigma_x G^{r,<} \sigma_x + i a_3 [\sigma_x, G^{r,<}] \}. \quad (6)$$

Here  $a_i$  ( $i = 1, 2, 3$ ) are the factors associated with the directions of the momenta,  $a_1 = 1 + \cos(\phi_{\mathbf{p}} - \phi_{\mathbf{k}})$ ,  $a_2 = 1 - \cos(\phi_{\mathbf{p}} - \phi_{\mathbf{k}})$ ,  $a_3 = \sin(\phi_{\mathbf{p}} - \phi_{\mathbf{k}})$ .

In order to investigate the transport properties of 2D system driven by a dc electric field  $\mathbf{E}$  along  $x$  axis, the dominant task is to carry out the nonequilibrium less Green's functions  $\hat{G}^<$ . The measurable quantities such as current and spin-current, are closely related to them. It is well known that the electric current operator in the spin basis reads  $\mathbf{J} = \sum_{\mathbf{p}} \hat{\Psi}^+ \mathbf{j} \hat{\Psi}$ , with the one-particle matrix current operator  $\mathbf{j}$  being  $\mathbf{j} = e \left[ \frac{\mathbf{p}}{m} + \alpha(\mathbf{n} \times \vec{\sigma}) \right]$ . The current operator of the  $i$ -direction spin can be defined through the electric current operator:  $\mathbf{J}^i = (\mathbf{J} \sigma_i + \sigma_i \mathbf{J})/4e$ .<sup>17</sup> We find, in the helicity basis, the operator of  $z$ -direction-spin current along  $y$  axis is nondiagonal. By taking the statistical average, the Hall spin-current can be computed via

$$J_y^z = \sum_{\mathbf{p}} \frac{p_y}{2m} [\hat{\rho}_{12}(\mathbf{p}) + \hat{\rho}_{21}(\mathbf{p})], \quad (7)$$

with the distribution functions  $\hat{\rho}_{\alpha\beta}(\mathbf{p}) = -i \int \frac{d\omega}{2\pi} \hat{G}_{\alpha\beta}^<(\mathbf{p}, \omega) (\alpha, \beta = 1, 2)$ . It can be clearly seen that the presence of spin-Hall effect is due to the nonvanishing of the  $\hat{\rho}_{12}$  and  $\hat{\rho}_{21}$ , viz. interband polarizations.

We follow the standard procedures described in Ref. 18 to derive the kinetic equations. We first construct the equations for the less Green's functions in the spin basis, and then modify it by the local unitary transformation. Finally, for steady and homogeneous systems, we arrive at the kinetic equations in the helicity basis

$$\begin{aligned} ie\mathbf{E} \cdot \left( \nabla_{\mathbf{p}} \hat{G}^< + \frac{i \nabla_{\mathbf{p}} \phi_{\mathbf{p}}}{2} [\hat{G}^<, \sigma_x] \right) - \alpha p B^< \\ = (\hat{\Sigma}^r \hat{G}^< - \hat{G}^< \hat{\Sigma}^a - \hat{G}^r \hat{\Sigma}^< + \hat{\Sigma}^< \hat{G}^a) \end{aligned} \quad (8)$$

with  $B^< = [\hat{G}^<, \sigma_z]_-$ . In this formula, the arguments of the Green's functions  $(\mathbf{p}, \omega)$  are dropped for shortness. In deriving the equations, the scalar potential gauge is used and only the lowest order of gradient expansion is taken into account. Noted that the second term in the bracket of the left hand of Eq. (8) arises from the fact that the transformation is local. As we will show below, it will lead to the intrinsic spin-Hall conductivity. At the same time, when the electron-impurity collision is chosen to be short-ranged, these kinetic equations reduce to the formula obtained in Ref. 13 by means of the Keldysh formalism.

We are only concerned with the linear response of the system to dc fields. Hence, the kinetic equations can be linearized with respect to the external electric fields. We rewrite the less Green's functions as  $\hat{G}^< = \hat{G}_0^< + \hat{G}_1^<$ , with  $\hat{G}_0^<$  and  $\hat{G}_1^<$  being in the zero- and first-order of  $E$ , respectively. As a result, the kinetic equations for the less Green's functions  $\hat{G}_1^<$  have the forms

$$\begin{aligned} -\alpha p \hat{C}_1 + ie\mathbf{E} \cdot \nabla_{\mathbf{p}} \hat{G}_0^< - \frac{1}{2} e\mathbf{E} \cdot \nabla_{\mathbf{p}} \phi_{\mathbf{p}} \hat{D}_0 = \\ (\hat{\Sigma}^r \hat{G}^< - \hat{G}^< \hat{\Sigma}^a - \hat{G}^r \hat{\Sigma}^< + \hat{\Sigma}^< \hat{G}^a)_1, \end{aligned} \quad (9)$$

where the matrices  $\hat{C}_1$  and  $\hat{D}_0$  are

$$\hat{C}_1 = \begin{pmatrix} 0 & -2(\hat{G}_1^<)_{12} \\ 2(\hat{G}_1^<)_{21} & 0 \end{pmatrix}, \quad (10)$$

$$\hat{D}_0 = \begin{pmatrix} 0 & (\hat{G}_0^<)_{11} - (\hat{G}_0^<)_{22} \\ (\hat{G}_0^<)_{22} - (\hat{G}_0^<)_{11} & 0 \end{pmatrix}. \quad (11)$$

The subscript 1 in the right-hand side of Eq. (9) stands for keeping this term in the first-order of  $E$ .

From Eq. (9) we can see that two driving terms enter the kinetic equations:  $ie\mathbf{E} \cdot \nabla_{\mathbf{p}} \hat{G}_0^<$  and  $-e\mathbf{E} \cdot \nabla_{\mathbf{p}} \phi_{\mathbf{p}} \hat{D}_0/2$ .

Hence we can assume that the solutions of these equations are formed from two terms  $\hat{G}_1^{(1)}$  and  $\hat{G}_1^{(2)}$ . In fact, when the disorder becomes short-ranged, these two parts of solutions connect with the terms in Kubo formalism. The first part of solutions stems from the driving term completely and corresponds to the bubble diagram in the scheme of Kubo formula.<sup>10,11,12</sup> At the same time, the another part of solutions, relating to the electron-impurity scattering, should be understood as the vertex correction. In this way, the distribution functions in the first order of dc field,  $\hat{\rho}_1(t)$ , can be written as  $\hat{\rho}_1(t) = \hat{\rho}_1^{(1)}(t) + \hat{\rho}_1^{(2)}(t)$  and, correspondingly, the spin-Hall effect comes from two different interband-polarization processes.

The less Green's functions  $\hat{G}_1^{(1)}$ , associated with the matrix  $\hat{D}_0$ , relate to the function  $n_F(\omega)$ . It is a non-diagonal matrix with same elements. In fact,  $\hat{G}_1^{(1)}$  originate from the nondiagonal elements of linear response retarded Green's functions,  $(\hat{G}_1^{(1)})_{12} = (\hat{G}_1^{(1)})_{21} = -2in_F \text{Im}(\hat{G}_1^r)_{12}$ . The latter one,  $(\hat{G}_1^r)_{12}$ , obeys the Dyson equation

$$2\alpha p(\hat{G}_1^r)_{12} + \frac{1}{2p}eE \sin \phi_{\mathbf{p}}[(\hat{G}_0^r)_{11} - (\hat{G}_0^r)_{22}] = (\hat{G}_1^r)_{12}[(\hat{\Sigma}_0^r)_{11} - (\hat{\Sigma}_0^r)_{22}] - (\hat{\Sigma}_1^r)_{12}[(\hat{G}_0^r)_{11} - (\hat{G}_0^r)_{22}], \quad (12)$$

and can be obtained analytically as

$$(\hat{G}_1^r)_{12} = -\frac{eE}{4\alpha p^2} \sin \phi_{\mathbf{p}}[(\hat{G}_0^r)_{11} - (\hat{G}_0^r)_{22}]. \quad (13)$$

It is well known that the linear response can not disturb the retarded Green's functions in semiconductors without spin-orbit interaction.<sup>19</sup> However, when the spin-orbit coupling is introduced, even weak dc field can produce the transition between the spin-orbit-coupled bands, resulting in interband polarization. The factor  $n_F(\omega)$  in the less Green's functions  $\hat{G}_1^{(1)}$  indicates that all electrons join in this process.

Substituting the Eq. (13) into (7) and neglecting the broadening of the noninteracting retarded Green's func-

tions, we find the contribution from the first polarization process to the spin-Hall conductivity

$$\sigma_{sH}^{(1)} = \frac{-e}{16\pi m\alpha} \int_0^\infty dp n_F(\varepsilon_1(p) - \mu) - n_F(\varepsilon_2(p) - \mu), \quad (14)$$

where  $\mu$  is the chemical potential. At zero temperature,  $\sigma_{sH}^{(1)}$  approaches the ballistic value  $e/8\pi$ . Note that this formula agrees with that obtained in the previous studies.<sup>7,11,12</sup>

The remaining part of Eq. (9) describes the transport process and is associated with the function  $\partial n_F(\omega)/\partial \omega$ . To solve it, we employ the two-band generalized Kadanoff-Baym ansatz (GKBA). This ansatz is widely used in the quantum kinetics of semiconductors driven by ac fields<sup>20,21</sup> and accurate enough to yield the quantitative agreements with experiments.<sup>18</sup> To study the linear response of 2DEG with SO coupling, we only need to consider the GKBA in the first order of dc field strength  $E$ . To further simplify the treatment, we ignore the broadening of  $\hat{G}_0^{r,a}$  induced by electron-impurity scattering. It is well known, this approximation combined with the gradient approximation are referred as the Boltzmann limit and valid when the spatial and temporal variations change slowly.<sup>18</sup> They are effective for modelling the quasichlassical optical and transport processes in semiconductors.

By taking into account the time-independence of  $\hat{\rho}(t)$  in the steady-state transport, the second part of solution of Eq. (9) can be obtained analytically. Retaining the results to  $O(n_i)$ , we find that the functions  $(\hat{\rho}_1^{(2)})_{12}$  can be expressed as

$$(\hat{\rho}_1^{(2)})_{12}(\mathbf{p}) = \zeta(p)eE \sin \phi_{\mathbf{p}} + i\xi(p)eE \sin \phi_{\mathbf{p}}, \quad (15)$$

where  $\zeta(p)$  and  $\xi(p)$  are real functions. Only the real part of  $(\hat{\rho}_1^{(2)})_{12}$  needs to be calculated, since the contribution of the imaginary part to the spin Hall current vanishes due to the symmetry relation  $(\hat{\rho}_1^{(2)})_{12} = (\hat{\rho}_1^{*(2)})_{21}$ .

The elementary calculation yields

$$\begin{aligned} \zeta(p) = & \frac{1}{4\alpha p} \left\{ \nabla_E n_F(E) \Big|_{E=\varepsilon_2(p)} (\alpha + p/m) - \nabla_E n_F(E) \Big|_{E=\varepsilon_1(p)} (-\alpha + p/m) \right. \\ & \left. + \frac{1}{\tau_{221}} [\Theta_2(p) + \Theta_1(p + 2m\alpha)] (\alpha + p/m) - \frac{1}{\tau_{212}} [\Theta_1(p) + \Theta_2(p - 2m\alpha)] (-\alpha + p/m) \right\}. \end{aligned} \quad (16)$$

The functions  $\Theta_1(p)$  and  $\Theta_2(p)$ , coupled by the equations,

$$-\nabla_E n_F(E) \Big|_{E=\varepsilon_\mu(p)} = \frac{\Theta_\mu(p)}{\tau_{1\mu\mu}} + \frac{\Theta_\mu(p)}{\tau_{2\mu\bar{\mu}}} - \frac{\Theta_{\bar{\mu}}(p + (-1)^\mu 2m\alpha)}{\tau_{3\mu\bar{\mu}}}, \quad (17)$$

connect to the diagonal distribution functions

$$(\hat{\rho}_1)_{\mu\mu}(\mathbf{p}) = [(-1)^\mu \alpha + p/m] \Theta_\mu(p) eE \cos \phi_{\mathbf{p}}, \quad (18)$$

with  $\mu = 1, 2$  and  $\bar{\mu} = 3 - \mu$ . In these equations, several

different relaxation times, defined by

$$\frac{1}{\tau_{i\mu\nu}} = 2\pi n_i \sum_k |V(\mathbf{p} - \mathbf{k})|^2 \Lambda_{i\mu\nu}(\phi_{\mathbf{k}-\mathbf{p}}, p, k), \quad (19)$$

emerge due to the long-range potential. Here the  $\Lambda_{i\mu\nu}$  are  $\Lambda_{1\mu\nu}(\phi, p, k) = \frac{1}{2} \sin^2 \phi \delta(\varepsilon_{\mu p} - \varepsilon_{\nu k})$ ,  $\Lambda_{2\mu\nu}(\phi, p, k) = \frac{1}{2}(1 - \cos \phi) \delta(\varepsilon_{\mu p} - \varepsilon_{\nu k})$  and  $\Lambda_{3\mu\nu}(\phi, p, k) = \frac{1}{2} \cos \phi (1 - \cos \phi) \delta(\varepsilon_{\mu p} - \varepsilon_{\nu k})$ .

The contribution of second interband polarization process to the spin-Hall conductivity can be evaluated by

$$\sigma_{sH}^{(2)} = \frac{e}{4\pi} \int_0^\infty \frac{p^2}{m} \zeta(p) dp. \quad (20)$$

It should be noted that the spin-Hall conductivity coming from the terms in the second line of Eq. (16) is zero at zero temperature. This is straightforwardly deduced from the fact that zero-temperature difference between the Fermi momenta for the two spin-orbit-coupled bands is  $2m\alpha$  and the factor  $\nabla_E n_F(E)$  reduces to the delta function. Omitting these terms at nonzero temperature is also supported by numerical estimation. At temperature lower than 4.2 K, considering long-range collision between the remote impurities and electrons in GaAs/AlGaAs heterojunctions,<sup>22</sup> we find the magnitude of contributions of these terms to the spin-Hall conductivity more than five orders smaller than that from the other terms. Hence, the spin-Hall conductivity  $\sigma_{sH}^{(2)}$  can be simplified as

$$\sigma_{sH}^{(2)} = \frac{e}{16\pi m\alpha} \int_0^\infty p dp \left\{ \nabla_E n_F(E) \big|_{E=\varepsilon_2(p)} (\alpha + p/m) - \nabla_E n_F(E) \big|_{E=\varepsilon_1(p)} (-\alpha + p/m) \right\}. \quad (21)$$

Finally, we obtain  $\sigma_{sH}^{(2)} = -\sigma_{sH}^{(1)}$ .

From the Eqs. (16) and (17) we can see that the spin-Hall conductivity  $\sigma_{sH}^{(2)}$  appears from the transport pro-

cess. In the dilute impurity limit, the diagonal distribution functions of nonequilibrium electrons driven by dc fields are of order of  $n_i^{-1}$ . These perturbed electrons in one band can transit to another band, resulting in interband polarization. The latter process is proportional to  $n_i$  and, therefore, the nondiagonal distribution functions in the lowest order of  $n_i$  appear. Hence, it is obvious that, here, the disorder plays an intermediate role. Such disorder-mediated process, in contrast to the one produced by all electrons, contributes from the electrons near the Fermi surfaces.

Note that in our treatment the influence of broadening of Green's functions on the spin-Hall effect remains untouched. This implies that our results are useful for the relative clean samples. At the same time, the derived kinetic equations in this letter can serve as the start point for further studying the effect of collision broadening, which is expected to affect on the linear response of the retarded and less Green's functions in different fashions. Hence, the spin-Hall effect may be observed in dirty samples.

In conclusion, we have presented a nonequilibrium Green's functions approach to the spin-Hall effect in Rashba two-dimensional electron systems. By taking into account the long-range scattering between impurities and electrons, the intrinsic spin-Hall conductivity has been recovered. It has also been illustrated that the interband transition of nonequilibrium electrons, participating in longitudinal transport, results in a disorder-mediated spin-Hall effect. As the result, the total spin-Hall effect disappears.

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